New Types of Interactions between Solitons of the (2+1)-Dimensional Broer-Kaup-Kupershmidt Equation

Chao-Qing Dai, Jian-Ping Meng, and Jie-Fang Zhang

Institute of Nonlinear Physics, Zhejiang Normal University, Jinhua 321004, P. R. China

Reprint requests to C.-Q. D.; E-mail: dcq424@126.com

Z. Naturforsch. **60a,** 687 – 695 (2005); received August 5, 2005

By means of the extended homogeneous balance method and the variable separation approach, more general variable separation solutions of the (2+1)-dimensional Broer-Kaup-Kupershmidt equation are obtained. Based on the variable separation solution and by selecting appropriate functions, new types of interactions between the multi-valued and the single-valued solitons, such as compacton-like semi-foldon and compacton, peakon-like semi-foldon and peakon, and bell-like semi-foldon and dromion, are investigated. Meanwhile, we also discuss the phase shift of these interactions. — PACS: 02.30.Jr, 02.30.Ik

Key words: New Types of Interactions between Solitons; (2+1)-Dimensional BKK System; Extended Homogeneous Balance Method.

1. Introduction

Since Lou and Lu successfully applied a kind of variable separation method, now called the multilinear variable separation approach (MLVSA), firstly in 1995 for the Davey-Stewartsen (DS) equation [1], the MLVSA has been a nearly systematic process to solve (2+1)-dimensional nonlinear evolution systems. Recently, a quite "universal" formula has been found [1 – 4]

$$u \equiv \frac{\lambda(a_0a_3 - a_1a_2)P_xQ_y}{(a_0 + a_1P + a_2Q + a_3PQ)^2}, \tag{1}$$

which is valid for the suitable physical fields or potentials of various (2+1)-dimensional physically interesting nonlinear models, such as the DS equation, the asymmetric DS equation, the long dispersive wave (LDW) equation, the dispersive long wave (DLW) equation, the Broer-Kaup-Kupershmidt (BKK) system, the Nizhnik-Novikov-Veselov (NNV) equation, the asymmetric NNV system, the generalized NNV equation, the long wave-short wave interaction model, the Maccari system, the (2+1)-dimensional Ablowitz-Kaup-Newell-Segur (AKNS) system, a nonintegrable (2+1)-dimensional Korteweg-de Vries (KdV) equation, the general $(N+M)$-component AKNS system and the (2+1)-dimensional sine-Gordon equation. In expression (1), $P \equiv P(x,t)$ is an arbitrary function of $\{x,t\}$, $Q \equiv Q(y,t)$ may be either an arbitrary function of $\{y,t\}$ or an arbitrary solution of a Riccati equation, while λ, a_0, a_1, a_2 and a_3 are taken as constants. In usual cases, the constant $\lambda = \pm 2$ or $\lambda = \pm 1$.

Nowadays the MLVSA is still in progress aiming at two aspects. One is to extend this method to other nonlinear systems, such as the differential-difference equations [5] and the (1+1)-dimensional nonlinear systems [6]. Another is to obtain more general solutions in the sense that it admits more arbitrary separation functions into the solutions, and to search more new soliton structures. Apart from the traditional localized excitations including lumps, dromions, peakons, compactons, foldons, ring solitons, fractal solitons, chaotic solitons and so on [1 – 4], the properties of dromion-dromion, peakon-peakon, compacton-compacton, foldon-foldon interactions were studied [2 – 4]. More recently, the properties of dromion-compacton, peakon-compacton, dromion-peakon interactions [7] were discussed. From mentioned above, we know that the interactions were discussed either between single-valued solitons, or between multi-valued solitons (foldons). To our best knowledge, the interactions between multi-valued and single-valued solitons, such as compacton-like semi-foldon and compacton, peakon-like semi-foldon and peakon, and bell-like semi-foldon and dromion, which are focused in the present paper, were little reported in previous literature.

In order to study these new types of interactions, we take the (2+1)-dimensional Broer-Kaup-Kupershmidt equation (BKKE)

$$H_{ty}-H_{xxy}+2(HH_x)_y+2G_{xx}=0, \tag{2}$$

$$G_t+G_{xx}+2(HG)_x=0 \tag{3}$$

as a concrete example. The (2+1)-dimensional BKKE may be derived from the inner parameter-dependent symmetry constraint of the Kadomtsev-Petviashvili (KP) equation [8]. Using some suitable dependent and independent variable transformations, Chen and Li have proved that the (2+1)-dimensional BKKE can be transformed to the (2+1)-dimensional integrable dispersive long wave equation (DLWE) [8]

$$u_{ty}=-\eta_{xx}-\frac{1}{2}(u^2)_{xy}, \tag{4}$$

$$\eta_t=-(u\eta+u+u_{xy})_x, \tag{5}$$

and the (2+1)-dimensional integrable Ablowitz-Kaup-Newell-Segur equation (AKNSE)

$$\psi_t=-\psi_{xx}+\psi u, \tag{6}$$

$$\phi_t=\phi_{xx}-\phi u, \tag{7}$$

$$u_y=\psi\phi. \tag{8}$$

When we take $y=x$, the (2+1)-dimensional BKKE is reduced to the usual (1+1)-dimensional BKKE, which can be used to describe the propagation of long wave in shallow water [9].

In [11], using the extended homogeneous balance method [12], we have proved that the (2+1)-dimensional BKKE possesses a general variable separation solution (1). Zhu et al. [13] presented a more general solution for the (2+1)-dimensional BKKE. In this paper, we extend these results in [11, 13] to a more general one with more arbitrary functions. Meanwhile, based on the "universal" formula (1), some new types of interactions between compacton-like semi-foldon and compacton, peakon-like semi-foldon and peakon, and bell-like semi-foldon and dromion will be discussed.

2. Variable Separation Solutions for the (2+1)-Dimensional BKKE

According to the extended homogeneous balance method, we assume (2) and (3) have the solutions

$$H=f'(w)w_x+u_0, \tag{9}$$
$$G=g''(w)w_xw_y+g'(w)w_{xy}+v_0, \tag{10}$$

where $f(w)$, $g(w)$, $w(x,y,t)$ are to be determined later, and u_0, v_0 are two seed solutions. It is evident that (2) and (3) possess trivial seed solutions $u_0=u_0(x,t)$, $v_0=0$.

Now substituting (9) and (10) together with the seed solutions into (2) and (3) yields

$$\begin{aligned}&H_{ty}-H_{xxy}+2(HH_x)_y+2G_{xx}=\\&\big(2f''^2+2f'f^{(3)}-f^{(4)}+2g^{(4)}\big)w_x^3w_y+f^{(3)}\big(w_xw_yw_t-3w_x^3w_{xy}-3w_xw_yw_{xx}+2u_0w_x^2w_y\big)\\&+f''\big(w_{xy}w_t+w_xw_{ty}+w_yw_{xt}-3w_{xy}w_{xx}-3w_xw_{xxy}-w_yw_{xxx}+4u_0w_xw_{xy}+2u_0w_{xx}w_y+2u_{0x}w_xw_y\big)\\&+f'\big(w_{xyt}-w_{xxxy}+2u_0w_{xxy}+2u_{0x}w_{xy}\big)+2g'w_{xxxy}+g^{(3)}\big(6w_xw_{xx}w_y+6w_x^2w_{xy}\big)\\&+g''\big(2w_{xxx}w_y+6w_{xx}w_{xy}+6w_xw_{xxy}\big)+f'f''\big(6w_xw_{xx}w_y+4w_x^2w_{xy}\big)+f'^2\big(2w_{xx}w_{xy}+2w_xw_{xxy}\big)=0,\end{aligned} \tag{11}$$

$$\begin{aligned}&G_t+G_{xx}+2(HG)_x=\\&\big(g^{(4)}+2g^{(3)}f'+2g''f''\big)w_x^3w_y+g^{(3)}\big(3w_xw_{xx}w_y+3w_x^2w_{xy}+w_xw_yw_t+2u_0w_x^2w_y\big)\\&+g''\big(w_{xxx}w_y+3w_{xx}w_{xxy}+w_{xy}w_{yt}+w_yw_{xt}+2u_0w_{xx}w_y+4u_0w_xw_{xy}+2u_{0x}w_xw_y\big)\\&+g'\big(w_{xyt}+w_{xxxy}+2u_0w_{xxy}+2u_{0x}w_{xy}\big)+f'g''\big(4w_xw_{xx}w_y+4w_x^2w_{xy}\big)+f''g'w_x^2w_{xy}\\&+f'g'\big(w_{xx}w_{xy}+2w_xw_{xxy}\big)=0.\end{aligned} \tag{12}$$

Setting the coefficients of the term $w_x^3w_y$ in (11) and (12) to zero, we obtain two ordinary differential equations for functions $f(w)$ and $g(w)$:

$$2f''^2+2f'f^{(3)}-f^{(4)}+2g^{(4)}=0, \tag{13}$$

$$g^{(4)}+2g^{(3)}f'+2g''f''=0. \tag{14}$$

The following special solutions exist for (13) and (14):

$$f(w)=g(w)=\ln(w). \tag{15}$$

Therefore

$$f'f'' = -\frac{1}{2}f''', \quad f'^2 = -f''. \tag{16}$$

Using these results, (11) and (12) can be simplified as

$$\begin{aligned} &f'(w_t + w_{xx} + 2u_0 w_x)_{xy} \\ &+ f''\big[w_{xy}(w_t + w_{xx} + 2u_0 w_x) \\ &\quad + w_x(w_t + w_{xx} + 2u_0 w_x)_y \\ &\quad + w_y(w_t + w_{xx} + 2u_0 w_x)_x\big] \\ &+ f^{(3)}\big[w_x w_y(w_t + w_{xx} + 2u_0 w_x)\big] = 0, \end{aligned} \tag{17}$$

$$\begin{aligned} &g'(w_t + w_{xx} + 2u_0 w_x)_{xy} \\ &+ g''\big[w_{xy}(w_t + w_{xx} + 2u_0 w_x) \\ &\quad + w_x(w_t + w_{xx} + 2u_0 w_x)_y \\ &\quad + w_y(w_t + w_{xx} + 2u_0 w_x)_x\big] \\ &+ g^{(3)}\big[w_x w_y(w_t + w_{xx} + 2u_0 w_x)\big] = 0. \end{aligned} \tag{18}$$

Setting the coefficients of $f^{(3)}$, f'', f' and $g^{(3)}$, g'', g' in (17) and (18) to zero yields a set of partial differential equations for $w(x,y,t)$:

$$(w_t + w_{xx} + 2u_0 w_x)_{xy} = 0, \tag{19}$$

$$\begin{aligned} &w_{xy}(w_t + w_{xx} + 2u_0 w_x) \\ &+ w_x(w_t + w_{xx} + 2u_0 w_x)_y \\ &+ w_y(w_t + w_{xx} + 2u_0 w_x)_x = 0, \end{aligned} \tag{20}$$

$$w_x w_y(w_t + w_{xx} + 2u_0 w_x) = 0. \tag{21}$$

By analysis, we find that (19) – (21) are satisfied automatically under the condition

$$w_t + w_{xx} + 2u_0 w_x = 0. \tag{22}$$

From mentioned above, (2) and (3) are linearized by the transformations (9) and (10).

Since the (22) is only a linear equation, one can certainly utilize the linear superposition theorem. For instance

$$w = \sum_{k=1}^{N} w_k(x,y,t) = Q_0(y) + \sum_{k=1}^{N} P_k(x,t)Q_k(y,t), \tag{23}$$

where the variable separated functions $P_k(x,t) \equiv P_k$ and $Q_k(x,t) \equiv Q_k$ $(k = 1,2,\cdots,N)$ are only the functions of $\{x,t\}$ and $\{y,t\}$, respectively, and $Q_0(y) \equiv Q_0$. Inserting the ansatz (23) into (22) yields the following simple variable separated equations:

$$P_{kt} + P_{kxx} + 2u_0 P_{kx} + b(t)P_k = 0, \tag{24}$$

$$Q_{kt} - b(t)Q_k = 0, \tag{25}$$

where $b(t)$ is an arbitrary function of indicated variable. Then the general variable separation solution for the BKKE yields

$$H = \frac{\sum_{k=1}^{N} P_{kx}Q_k}{Q_0 + \sum_{k=1}^{N} P_k Q_k} + u_0(x,t), \tag{26}$$

$$G = \frac{\sum_{k=1}^{N} P_{kx}Q_{ky}}{Q_0 + \sum_{k=1}^{N} P_k Q_k} - \frac{\sum_{k=1}^{N} P_{kx}Q_k[Q_{0y} + \sum_{k=1}^{N} P_k Q_{ky}]}{[Q_0 + \sum_{k=1}^{N} P_k Q_k]^2}, \tag{27}$$

where u_0, P_k and Q_k satisfy (24) and (25).

In order to discuss new types of interactions between solitons, we have to leave the complex forms (26) and (27) and need to make some simplifications further. If $N = 3$, $Q_0 = a_0$, $Q_1 = a_1$, $Q_2 = a_2 Q$, $Q_3 = a_3 Q$, ($a_i =$ constants, $i = 0,\cdots,3$), $P_1 = P_3 = P$, $P_2 = 1$ with $P \equiv P(x,t)$ and $Q \equiv Q(y,t)$, then (23) – (25) change into

$$w = a_0 + a_1 P + a_2 Q + a_3 PQ, \tag{28}$$

$$P_t + P_{xx} + 2u_0 P_x + b(t)P = 0, \tag{29}$$

$$Q_t - b(t)Q = 0. \tag{30}$$

From (29) and (30), u_0 and $Q(y,t)$ can be solved, that is

$$u_0 = -\frac{1}{2P_x}[P_t + P_{xx} + b(t)P], \tag{31}$$

$$Q(y,t) = \varphi(y)\exp[\int^t b(\tau)\mathrm{d}\tau], \tag{32}$$

where $\varphi(y) \equiv \varphi$ is an arbitrary function of $\{y\}$.

Therefore, a special variable separation excitation of the (2+1)-dimensional BKKE is

$$H = \frac{(a_1 + a_3 Q)P_x}{a_0 + a_1 P + a_2 Q + a_3 PQ} - \frac{1}{2P_x}[P_t + P_{xx} + b(t)P], \tag{33}$$

$$G = \frac{(a_0 a_3 - a_1 a_2)P_x Q_y}{[a_0 + a_1 P + a_2 Q + a_3 PQ]^2}, \tag{34}$$

where P is an arbitrary function of $\{x,t\}$ and Q is given in (32). From (34), we can see that it possesses the same form of the "universal" formula (1) with $\lambda = 1$.

3. New Types of Interactions between Solitons of the (2+1)-Dimensional BKKE

Here we do not discuss the general form (27) of G, and only discuss some localized structures for G expressed by (34). Actually, even in this special situation, one still finds abundant localized excitations for the (2+1)-dimensional BKKE because of the arbitrariness of the functions P, φ and b included in (32)–(34). In [11], we have given out some coherent solutions for the field G such as dromions, conic solitons, lumps, oscillating dromions, breathers, and instantons in details. Ying and Lou have obtained some coherent structures of the (2+1)-dimensional BKKE by means of the truncated Painlevé expansion and the related Bäcklund [14]. Lou has also constructed (2+1)-dimensional compacton solutions and discussed their interaction behaviors [15]. Here we are interested in revealing some new types of interactions between solitons in the (2+1)-dimensional BKKE such as the interaction behaviors between compacton-like semi-foldon and compacton, peakon-like semi-foldon and peakon, and bell-like semi-foldon and dromion.

Based on the physical quantity (34), semi-folded localized structures can be discussed, when function Q is a single-valued function and P is selected via the relations

$$P_x = \sum_{i=1}^{N} \kappa_i(\zeta - d_i t), \quad x = \zeta + \sum_{i=1}^{N} \chi_i(\zeta - d_i t), \qquad P = \int^{\zeta} P_x x_{\zeta} \mathrm{d}\zeta, \tag{35}$$

where d_i ($i = 1,2,\ldots,N$) are arbitrary constants, κ_i and χ_i are localized excitations with the properties $\kappa_i(\pm\infty) = 0,\ \chi_i(\pm\infty) = $ constants. From (35), one can know that ζ may be a multi-valued function in some suitable regions of x by choosing the functions χ_i appropriately. Therefore, the function P_x, which is obviously an interaction solution of M localized excitations due to the property $\zeta|_{x\to\infty} \to \infty$, may be a multi-valued function of x in these areas; though it is a single-valued function of ζ. Actually, most of the known multi-loop solutions are special cases of (35). More properties of the multi-valued function under change of the degree of multivaluedness can be found in [14].

In order to discuss the interaction property of localized excitations related to the physical quantity (1) or (34), we first study the asymptotic behaviors of the localized excitations produced from the formula (1) when $t \to \infty$.

3.1. Asymptotic Behaviors of the Localized Excitations Produced from (1)

In general, if functions P [considering (35)] and Q are selected as multi-localized solitonic excitations with ($z_i \equiv \zeta - d_i t$)

$$P|_{t\to\mp\infty} = \sum_{i=1}^{N} P_i^{\mp}, P_i^{\mp}(z_i) \equiv P_i(\zeta - d_i t) \equiv \int \kappa_i \mathrm{d}x|_{z_i\to\mp\infty}, \tag{36}$$

$$Q|_{t\to\mp\infty} = \sum_{j=1}^{M} Q_j^{\mp}, Q_j^{\mp} \equiv Q_j(y - c_j t + \Delta_j^{\mp}), \tag{37}$$

where $\{P_i, Q_j\}\forall i$ and j are localized functions, then the physical quantity expressed by (1) delivers $M \times N$ (2+1)-dimensional localized excitations with the asymptotic behavior

$$u|_{t\to\mp\infty} \to \sum_{i=1}^{N}\sum_{j=1}^{M} \frac{(a_3 a_0 - a_1 a_2) P_{iz_i}^{\mp} Q_{jy}^{\mp}}{(1+\chi_{iz_i}^{\mp})[a_0 + a_1(P_i^{\mp}(z_i) + \tilde{P}_i^{\mp}) + a_2(Q_j^{\mp} + \tilde{Q}_j^{\mp}) + a_3(P_i^{\mp}(z_i) + \tilde{P}_i^{\mp})(Q_j^{\mp} + \tilde{Q}_j^{\mp})]^2} \equiv \sum_{i=1}^{N}\sum_{j=1}^{M} u_{ij}^{\mp}, \tag{38}$$

$$x|_{t\to\mp\infty} \to \zeta + \delta_i^{\mp} + \chi_i^{\mp}(z_i), \tag{39}$$

with

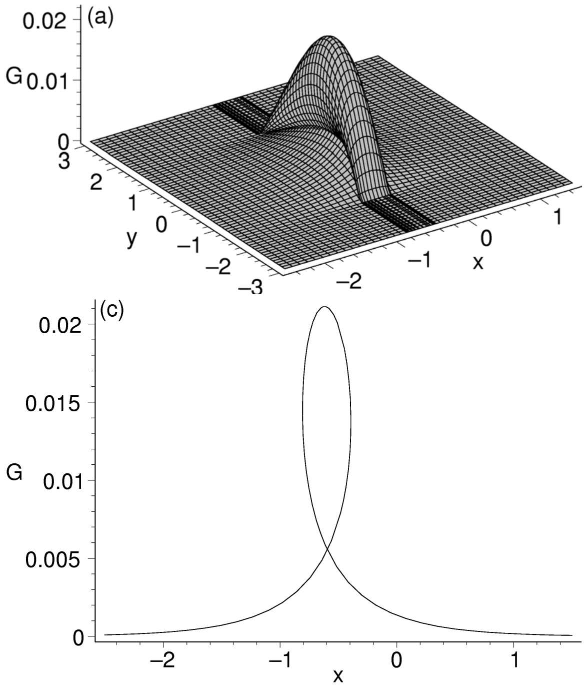
$$\tilde{P}_i^{\mp} = \sum_{j<i} P_j(\mp\infty) + \sum_{j>i} P_j(\pm\infty), \tag{40}$$

$$\tilde{Q}_i^{\mp} = \sum_{j<i} Q_j(\mp\infty) + \sum_{j>i} Q_j(\pm\infty), \tag{41}$$

$$\delta_i^{\mp} = \sum_{j<i} \chi_j(\mp\infty) + \sum_{j>i} \chi_j(\pm\infty). \tag{42}$$

In the above discussion, it has been assumed, without loss of generality, that $c_i > c_j$, $d_i > d_j$ if $i > j$. From the asymptotic result (38), we discover some important and interesting facts:

(i) The ij-th localized excitation u_{ij} is a travelling wave moving with the velocity d_i along the positive



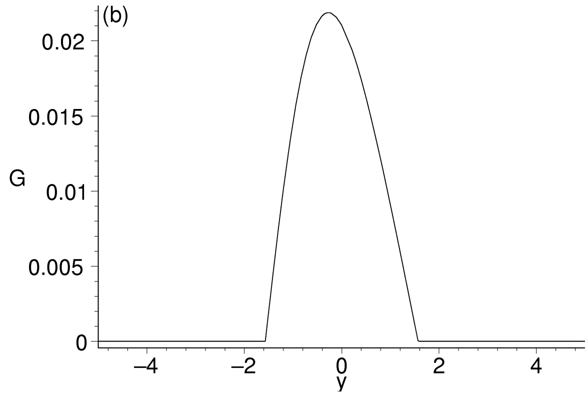


Fig. 1. Compacton-like semi-foldon structure for G with conditions (49) and (50) (a) at time $t = -2$; (b) a sectional view related to (a) at $x = -0.5$; (c) a sectional view related to (a) at $y = 0$.

($d_i > 0$) or negative ($d_i < 0$) x direction, and c_j along the positive ($c_j > 0$) or negative ($c_j < 0$) y direction.

(ii) The properties of the ij-th localized excitation u_{ij} is only determined by P_i of (36) and Q_j of (37).

(iii) The shape of the ij-th localized excitation u_{ij} will be changed (non-completely elastic or completely inelastic interaction) if

$$\tilde{P}_i^+ \neq \tilde{P}_i^-, \tag{43}$$

and (or)

$$\tilde{Q}_j^+ \neq \tilde{Q}_j^-, \tag{44}$$

following the interaction. On the contrary, it will preserve its shape (completely elastic interaction) during the interaction if

$$\tilde{P}_i^+ = \tilde{P}_i^-, \tag{45}$$

$$\tilde{Q}_j^+ = \tilde{Q}_j^-. \tag{46}$$

(iv) The phase shift of the ij-th localized excitation u_{ij} reads

$$\delta_i^+ - \delta_i^-, \tag{47}$$

in the x direction and

$$\Delta_j^+ - \Delta_j^-. \tag{48}$$

in the y direction.

3.2. Compacton-like Semi-foldon Structure

Due to the arbitraries of constants $a_0 \sim a_3$ in (34), we can discuss various localized excitations based on (34) by choosing special values of these constants $a_0 \sim a_3$ (see [2] in details) and the arbitrary functions P, φ and b. For discussing the following new types of semi-foldons and interaction behaviors, we take $a_0 = 8$, $a_1 = a_2 = 1$, $a_3 = 0.3$ for convenience.

Now we discuss some new coherent structures for the physical quantity G, and focus our attention on (2+1)-dimensional semi-folded localized structures, which may exist in certain intricate situations. When the function Q is a single-valued function and P is selected as (35) with $N = 1$, namely

$$P_x = 0.5\mathrm{sech}(\zeta - 0.3t)^2,$$

$$x = \zeta - 1.5\tanh(\zeta - 0.3t),$$

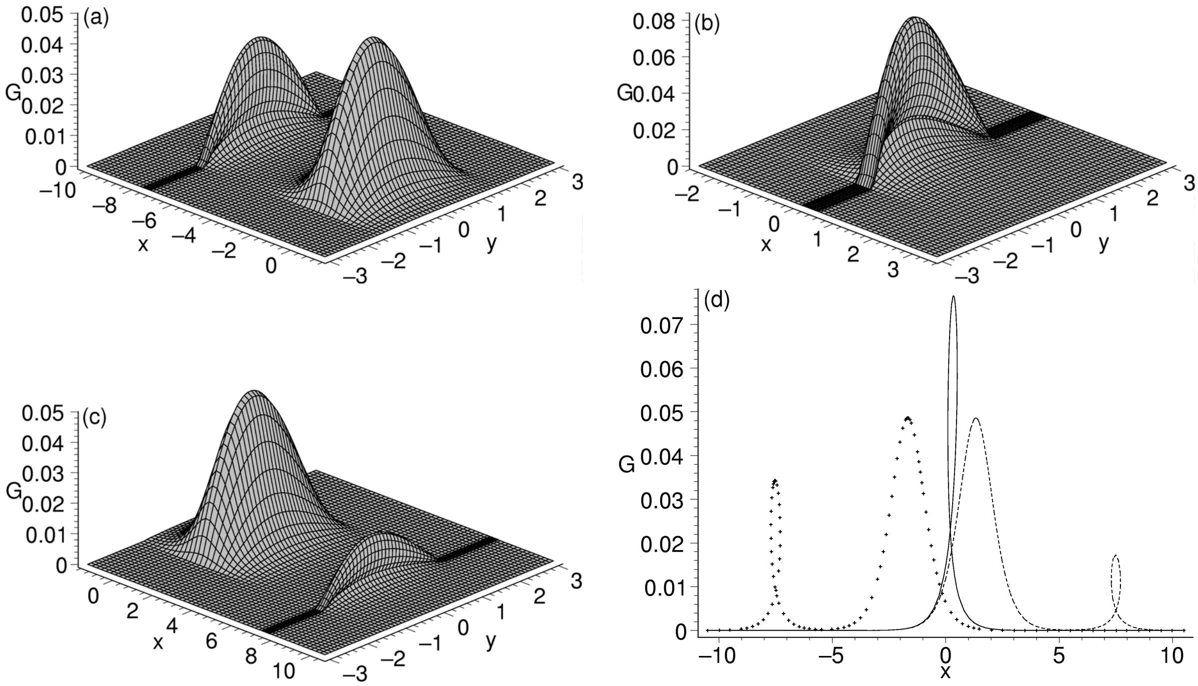


Fig. 2. Evolution profiles of the interaction between semi-foldon and compacton determined by (34) with (50) and (51) at (a) $t = -25$, (b) $t = 1$, (c) $t = 25$. (d) The corresponding sectional view at $\{t = -25, y = 0\}$ (dotted line before collision), $\{t = 1, y = 0\}$ (solid line in collision), $\{t = 25, y = 0\}$ (dashed line after collision), respectively.

$$P = \int^{\zeta} P_x x_{\zeta} \mathrm{d}\zeta, \tag{49}$$

$$Q = \begin{cases} 0, \quad y \le -\frac{\pi}{2}, \\ (\sin(y)+1)\exp(\mathrm{sech}(t)), \\ \quad -\frac{\pi}{2} < y \le \frac{\pi}{2}, \\ 2\exp(\mathrm{sech}(t)), \quad y > \frac{\pi}{2}, \end{cases} \tag{50}$$

then we can obtain a new type of semi-folded localized excitation looking like a compacton-like loop soliton (compacton-like semi-foldon). The corresponding coherent structure is depicted in Fig. 1, from which one can find that the semi-folded coherent structure possesses novel properties, namely, it folds as loop soliton in the x direction and localizes as compacton in the y direction.

3.3. Interaction between Compacton-like Semi-foldon and Compacton

Due to the arbitrariness of the functions in solution (34), we can discuss the interaction behavior between compacton-like semi-foldon and compacton by selecting Q as a single-valued piecewise smooth function (50) and P as (35) with $N = 2$, $\chi_2 = 0$, $d_1 = 0.3$, $d_2 = 0$, i.e.

$$P_x = 0.5\mathrm{sech}(\zeta - 0.3t)^2 + \mathrm{sech}(\zeta)^2,$$
$$x = \zeta - 1.5\tanh(\zeta - 0.3t), \quad P = \int^{\zeta} P_x x_{\zeta} \mathrm{d}\zeta. \tag{51}$$

From Fig. 2, one can see the interaction between multi-valued compacton-like semi-foldon and single-valued compacton is non-completely elastic since the amplitude of the moving semi-foldon decreases. This property is different from the completely elastic interaction between either single-valued and single-valued solitons [7, 15], or multi-valued and multi-valued solitons (foldons) [4]. Meanwhile, the phase shift can be observed. In order to reveal the phase shift more clearly and visually, it has proved convenient to fix the compacton possessing zero velocity. Prior to interaction, the compacton has set to be $\{v_{0x} = d_2 = 0, v_{0y} = 0\}$, however, the position located by compacton is still altered about from $x = -1.5$ to $x = 1.5$, then stops at $x = 1.5$ and preserves its initial velocities $\{v_x = v_{0x}, v_y = v_{0y}\}$ after interaction. Therefore the phase shift of the static compacton is $\delta_2^+ - \delta_2^- = \chi_1(-\infty) - \chi_1(+\infty) = 3$. The final velocities V_x and V_y of the mov-

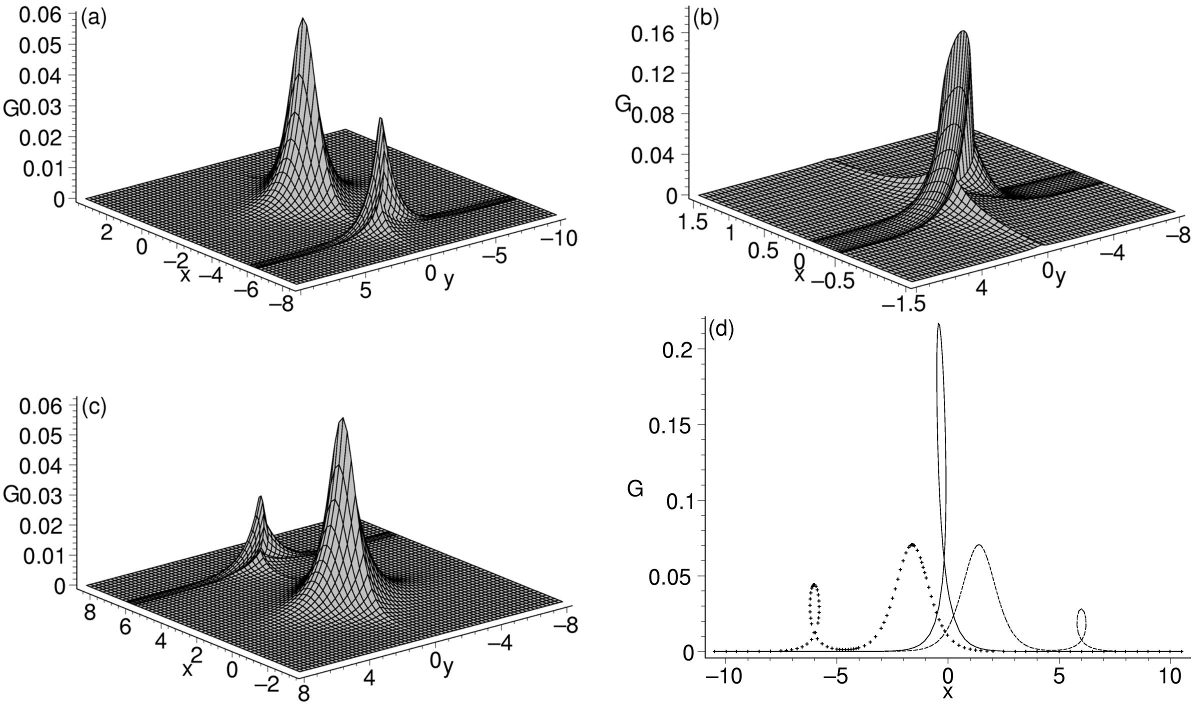


Fig. 3. Evolution profiles of the interaction between semi-foldon and peakon determined by (34) with (51) and (52) at (a) $t = -20$, (b) $t = -1$, (c) $t = 20$. (d) The corresponding sectional view at $\{t = -20, y = 0.01\}$ (dotted line before collision), $\{t = -1, y = 0.01\}$ (solid line in collision), $\{t = 20, y = 0.01\}$ (dashed line after collision), respectively.

ing compacton-like semi-foldon also completely preserved its initial velocities $\{V_x = V_{0x} = d_1 = 0.3, V_y = V_{0y} = 0\}$. The phase shift of the moving semi-foldon is $\delta_1^+ - \delta_1^- = \chi_2(+\infty) - \chi_2(-\infty) = 0$, that is, the for moving semi-foldon exists no phase shift.

3.4. *Interaction between Peakon-like Semi-foldon and Peakon*

Similarly, one of the simplest selections for discussing the interaction between peakon-like semi-foldon and peakon is to take an arbitrary multi-valued function and an arbitrary single-valued function, i.e. P is selected as (51) and Q has the following form

$$Q = \begin{cases} \exp(y)\exp(\mathrm{sech}(t)), & y < 0, \\ (-\exp(-y)+2)\exp(\mathrm{sech}(t)), & y \geq 0. \end{cases} \quad (52)$$

From Fig. 3, one can see the interaction behavior between peakon-like semi-foldon and peakon is also non-completely elastic. This semi-folded localized excitation folds as loop soliton in the x direction and localizes as peakon in the y direction; so we call it a peakon-like semi-foldon. The shape and amplitude of the static peakon hardly change after collision, while the amplitude of the moving semi-foldon with the velocity $\{V_x = 0.3, V_y = 0\}$ decreases. Before the interaction, the static peakon is located at $x = -1.5$ and after the interaction, it is shifted to $x = 1.5$. By careful analysis similar to Section 3.3, we know that the phase shift of the static peakon is 3, and for the moving semi-foldon exists no phase shift. These properties are analogous to the case in Section 3.3.

3.5. *Interaction between Bell-like Semi-foldon and Dromion*

Finally, we discuss the interaction between bell-like semi-foldon and dromion. When P possesses the form of (51) and Q is taken as

$$Q = \tanh(y)\exp(\mathrm{sech}(t)), \quad (53)$$

then the interaction between bell-like semi-foldon and dromion can be observed. This semi-folded localized excitation folds as loop soliton in the x direction and

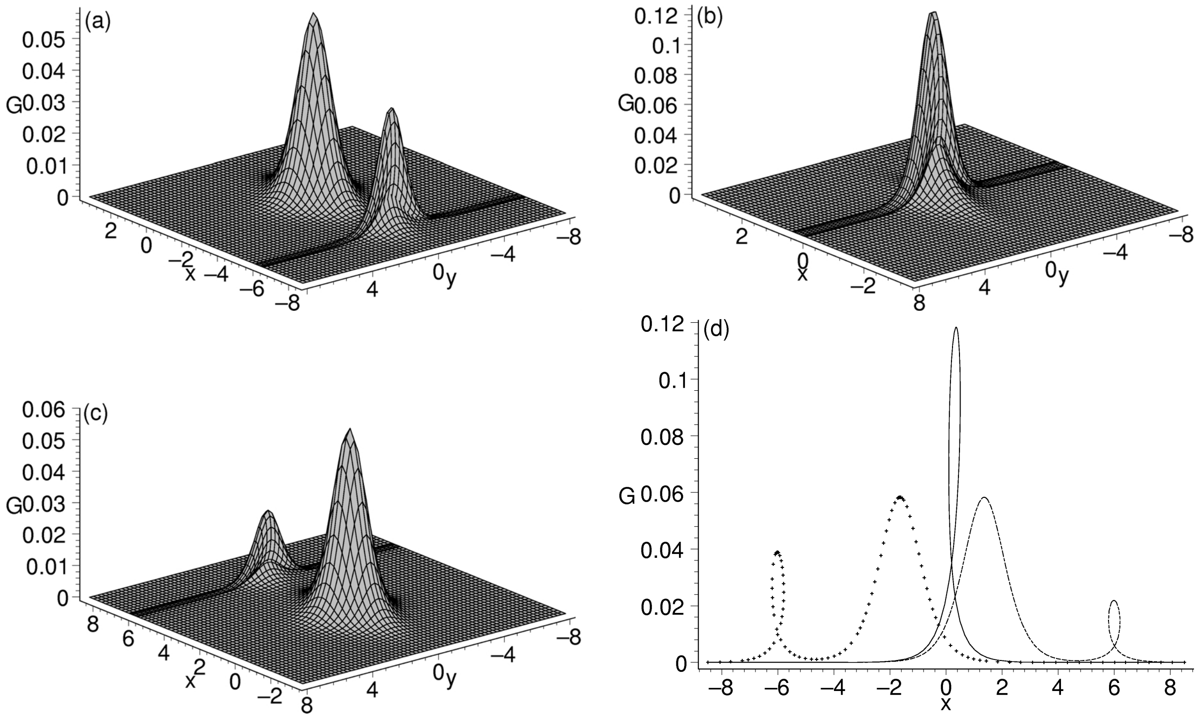


Fig. 4. Evolution profiles of the interaction between semi-foldon and dromion determined by (34) with (51) and (53) at (a) $t=-20$, (b) $t=1$, (c) $t=20$. (d) The corresponding sectional view at $\{t=-20, y=0\}$ (dotted line before collision), $\{t=1, y=0\}$ (solid line in collision), $\{t=20, y=0\}$ (dashed line after collision), respectively.

localizes like a bell in the y direction; so we call it a bell-like semi-foldon.

From Fig. 4, one can see the interaction between multi-valued semi-foldon and single-valued dromion is also non-completely elastic which is analogous to the above two cases. The shape and amplitude of the static dromion hardly change after collision, while the amplitude of the moving semi-foldon with the velocity $\{V_x = 0.3, V_y = 0\}$ decreases. Furthermore, the phase shift of the static dromion can be observed. Before the collision, the static dromion is located at $x=-1.5$ and after the collision, it is shifted to $x=1.5$. Similarly to analysis in Section 3.3, we know that the phase shift of the static dromion is 3, and for the moving semi-foldon exists no phase shift.

4. Summary and Discussion

In summary, using the extended homogeneous balance method and the variable separation approach, more general variable separation solutions of the (2+1)-dimensional Broer-Kaup-Kupershmidt equation are obtained. From the special variable separation solution (34) and by selecting appropriate functions, new types of interactions between the multi-valued and the single-valued solitons, such as compacton-like semi-foldon and compacton, peakon-like semi-foldon and peakon, and bell-like semi-foldon and dromion, are found. By means of analysis of asymptotic behaviors and the profiles in Figs. 2–4, one can find that their interactions are non-completely elastic which is different from the completely elastic interaction between either single-valued and single-valued solitons or multi-valued and multi-valued solitons (foldons). Moreover, the static single-valued solitons all exist phase shift, while the amplitudes of semi-foldons all decrease.

What we further verified that the extended homogeneous balance method and the variable separation approach are quite useful to generate abundant localized excitations for many models. In our future work, on the one hand, we devote to extend this method to other nonlinear systems, such as the differential-difference equations and (1+1)-dimensional nonlinear systems. On the other hand, we will look for more interesting localized excitations.

[1] S. Y. Lou and J. Z. Lu, J. Phys. A: Math. Gen. **29**, 4029 (1996).

[2] S. Y. Lou, Phys. Lett. A **277**, 94 (2000); X. Y. Tang and S. Y. Lou, Chaos, Solitons and Fractals **14**, 1451 (2002); S. Y. Lou, C. L. Chen, and X. Y. Tang, J. Math. Phys. **43**, 4078 (2002); S. Y. Lou, Phys. Scr. **65**, 7 (2002); X. Y. Tang, S. Y. Lou, and Y. Zhang, Phys. Rev. E **66**, 046601 (2002); X. Y. Tang and S. Y. Lou, Chaos, Solitons and Fractals **14**, 1451 (2002); X. Y. Tang, C. L. Chen, and S. Y. Lou, J. Phys. A: Math. Gen. **35**, L293 (2002); S. Y. Lou and H. Y. Ruan, J. Phys. A: Math. Gen. **30**, 305 (2001); S. Y. Lou, X. Y. Tang, and C. L. Chen, Chin. Phys. Lett. **19**, 769 (2002); K. Z. Hong, B. Wu, and X. F. Chen, Commun. Theor. Phys. **39**, 393 (2003); H. Y. Ruan and Y. X. Chen, Acta Phys. Sin. **50**, 586 (2001) (in Chinese); X. Y. Tang and S. Y. Lou, Commun. Theor. Phys. **38**, 1 (2002); H. Y. Ruan and Y. X. Chen, Acta Phys. Sin. **52**, 1313 (2003) (in Chinese); H. Y. Ruan and Y. X. Chen, Chaos, Solitons and Fractals **17**, 929 (2003); S. Y. Lou, J. Phys. A: Math. Gen. **36**, 3877 (2003); X. Y. Tang and S. Y. Lou, Chin. Phys. Lett. **20**, 335 (2003); K. Z. Hong and B. Wu, Commun. Theor. Phys. **39**, 393 (2003); J. F. Zhang, C. L. Zheng, J. P. Meng, and J. P. Fang, Chin. Phys. Lett. **20**, 653 (2003); J. F. Zhang, Chin. Phys. **10**, 89 (2001); J. F. Zhang, Commun. Theor. Phys. **38**, 277 (2002); J. F. Zhang, Chin. Phys. **11**, 651 (2002); J. F. Zhang, W. H. Huang, and C. L. Zheng, Acta Phys. Sin. **51**, 2627 (2002) (in Chinese); J. F. Zhang, Phys. Lett. A **313**, 410 (2003); C. L. Zheng and J. F. Zhang, Chin. Phys. Lett. **19**, 1399 (2002); C. L. Zheng, J. F. Zhang, and Z. M. Sheng, Chin. Phys. Lett. **20**, 331 (2003); J. F. Zhang, C. L. Zheng, J. P. Meng, and J. P. Fang, Chin. Phys. Lett. **20**, 448 (2003); C. L. Zheng, J. F. Zhang, W. H. Huang, and L. Q. Chen, Chin. Phys. Lett. **20**, 783 (2003); C. L. Zheng, J. M. Zhu, and J. F. Zhang, Commun. Theor. Phys. **39**, 261 (2003); J. F. Zhang and J. P. Meng, Chin. Phys. Lett. **20**, 1006 (2003); H. Y. Ruan and Y. X. Chen, Chaos, Solitons and Fractals **17**, 929 (2003).

[3] X. Y. Tang and S. Y. Lou, Chin. Phys. Lett. **20**, 335 (2003).

[4] X. Y. Tang and S. Y. Lou, Commun. Theor. Phys. **40**, 62 (2003); X. Y. Tang and S. Y. Lou, J. Math. Phys. **44**, 4000 (2003).

[5] X. M. Qian, S. Y. Lou, and X. B. Hu, J. Phys. A: Math. Gen. **37**, 2401 (2003); X. M. Qian, S. Y. Lou, and X. B. Hu, Z. Naturforsch. **59a**, 645 (2004).

[6] C. Z. Xu and J. F. Zhang, Commun. Theor. Phys. **42**, 568 (2004).

[7] J. F. Zhang, J. P. Meng, and W. H. Huang, Commun. Theor. Phys. **42**, 161 (2004).

[8] S. Y. Lou and X. B. Hu, J. Math. Phys. **38**, 6401 (1997).

[9] C. L. Chen and Y. S. Li, Commun. Theor. Phys. **38**, 129 (2002).

[10] V. E. Zakharov and J. Li, Appl. Mech. Tech. Phys. (USSR) **9**, 190 (1968).

[11] J. F. Zhang and P. Han, Acta Phys. Sin. **51**, 0705 (2002) (in Chinese).

[12] M. L. Wang, Phys. Lett. A **199**, 169 (1995); J. F. Zhang, Chin. Phys. Lett. **16**, 659 (1999).

[13] J. M. Zhu, Z. Y. Ma, and C. L. Zheng, Acta Phys. Sin. **53**, 3248 (2004) (in Chinese).

[14] J. P. Ying and S. Y. Lou, Z. Naturforsch. **56a**, 619 (2001).

[15] S. Y. Lou, J. Phys. A: Math. Gen. **35**, 10619 (2002).